

Higher-Spin Fields in Braneworlds

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The dynamics of higher-spin fields in braneworlds is discussed. In particular, we study fermionic and bosonic higher-spin fields in AdS_5 and their localization on branes. We find that four-dimensional zero modes exist only for spin-one fields, if there are no couplings to the boundaries. If boundary couplings are allowed, as in the case of the bulk graviton, all bosons acquire a zero mode irrespective of their spin. We show that there are boundary conditions for fermions, which generate chiral zero modes in the four-dimensional spectrum. We also propose a gauge invariant on-shell action with cubic interactions by adding non-minimal couplings, which depend on the Weyl tensor. In addition, consistent couplings between higher-spin fields and matter on the brane are presented. Finally, in the AdS/CFT correspondence, where bulk 5D theories on AdS are related to 4D CFT s, we explicitly discuss the holographic picture of higher-spin theories in AdS_5 with and without boundaries.

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1. INTRODUCTION

The problem of consistent higher-spin (HS) gauge theories is a fundamental problem in field theory. After the description of their free dynamics [1], [2], only negative results for their interactions were obtained [3],[4]. For example, it was realized that HS fields cannot consistently minimally interact with gravity. However, by allowing additional gaugings, one may introduce counter terms, which make the propagation of HS fields in curved backgrounds well-defined. By appropriate completion of the interactions, Vasiliev equations are found, which are the generally covariant field equations for massless HS gauge fields describing their consistent interaction with gravity [5],[6],[7].

Nowadays, there is a renewal interest in HS gauge theories. A basic reason for this is that HS theories exist on anti-de Sitter spaces AdS [8], signaling their relevance in the AdS/CFT correspondence. In this framework, as a general rule, conserved currents in the boundary CFT are expected to correspond to massless gauge fields in the bulk [9]. A weakly coupled boundary gauge theory for example contains an infinite number of almost conserved currents, which will be

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described by a dual HS gauge theory defined in the bulk of AdS . Although much remain to be done in this direction, specific progress has been made in three-dimensional CFT s. It was proposed in [10] for example, that the singlet sector of the three-dimensional critical $O(N)$ vector model is dual, in the large N limit, to a minimal theory in four-dimensional anti-de Sitter space containing massless gauge fields of even spin of the kind studied in [5]. String theory also gives additional support to HS fields. Indeed, string theory, contains an infinite number of massive HS fields with consistent interactions. In the low-tension limit, their masses disappear. Massless HS theories are thus the natural candidates for the description of the low-tension limit of string theory at the semi-classical level [11]. The hope is that the understanding of the dynamics of HS fields could help towards a deeper insight of string theory, which now is mainly based on its low-spin excitations and their low-energy interactions.

A generic massless bosonic particle of integer spin s in an n -dimensional spacetime is described by a totally symmetric tensor of rank s , $\phi_{\mu_1\mu_2\ldots\mu_s}$, while a fermionic particle of spin $s+\frac{1}{2}$ by a totally symmetric tensor-spinor of rank s , $\psi_{\mu_1\mu_2\ldots\mu_s}$. These fields are defined up to gauge transformations and they are subject to certain constraints such that the corresponding theories are ghost free. This means that they describe exactly two propagating modes of $\pm s$ and $\pm(s+\frac{1}{2})$ helicities, for bosons and fermions, respectively. Such theories may be obtained as the massless limits [1],[2] of massive HS theories [12] or by gauge invariance and supersymmetry, as the latter relates HS fields to known lower spin ones [13].

In flat Minkowski spacetime, the gauge transformations of the HS fields are

$$\delta\phi_{\mu_1\mu_2\ldots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\mu_3\ldots\mu_s)}, \quad \delta\psi_{\mu_1\mu_2\ldots\mu_s} = \partial_{(\mu_1}\epsilon_{\mu_2\mu_3\ldots\mu_s)}, \quad (1.1)$$

where the parenthesis denote the symmetrized sum of s -terms (without the usual combinatorial $s!$) and $\xi_{\mu_2\mu_3\ldots\mu_s}$, $\epsilon_{\mu_2\mu_3\ldots\mu_s}$ are totally symmetric rank- $(s-1)$ tensor and tensor-spinors, respectively. In addition, we impose on these fields the strongest gauge invariant constraints

$$\phi^{\mu\nu}_{\mu\nu\mu_5\ldots\mu_s} = 0, \quad \gamma^\nu\psi_{\nu\mu_4\ldots\mu_s} = 0, \quad (1.2)$$

which means that the bosonic HS fields are double traceless, while the fermionic ones are triple γ -traceless (as a trace in the fermionic conditions can be considered as due to two γ matrices). These conditions give constraints for $s \geq 4$ and $s \geq \frac{7}{2}$ for bosons and fermions, respectively and eliminate their lower-spin components. In addition, one can impose traceless and γ -traceless of the gauge parameters ξ, ϵ , respectively, i.e.,

$$\xi^\nu_{\nu\mu_4\ldots\mu_s} = 0, \quad \gamma^\nu\epsilon_{\nu\mu_3\mu_4\ldots\mu_s} = 0. \quad (1.3)$$

It should be noted, however, that there exists also a recently proposed formulation [14],[15], where the gauge parameters are not constrained.

A simple counting reveals that there are only two independent degrees of freedom for both the bosonic and fermionic HS fields. In this case, consistent ghost-free equations of motions for the massless gauge HS fields can be written down, which described the propagation of the two helicity modes of these fields [16],[17]. In particular, pure gauge degrees of freedom can be eliminated by imposing, for integer spins, the appropriate generalizations of the Lorentz and de Donder gauges

$$\partial_\nu\phi^\nu_{\mu_2\mu_3\ldots\mu_s} - \frac{1}{2}\partial_{(\mu_2}\phi^\nu_{\nu\mu_3\ldots\mu_s)} = 0, \quad (1.4)$$

whereas the corresponding gauge conditions for half-integer fields reads

$$\gamma^\nu\psi_{\nu\mu_2\mu_3\ldots\mu_s} - \frac{1}{2s}\gamma_{(\mu_2}\psi^\nu_{\nu\mu_3\ldots\mu_s)} = 0. \quad (1.5)$$

In this case, the bosonic $\phi_{\mu_1\mu_2\ldots\mu_s}$ and the fermionic $\psi_{\mu_1\mu_2\ldots\mu_s}$ fields satisfy

$$\square\phi_{\mu_1\mu_2\ldots\mu_s} = 0, \quad \not{D}\psi_{\mu_1\mu_2\ldots\mu_s} = 0. \quad (1.6)$$

Thus, $\phi_{\mu_1\mu_2\ldots\mu_s}$, $\psi_{\mu_1\mu_2\ldots\mu_s}$ indeed describe massless particles, as claimed.

It is clear from the above that there is no problem of writing down HS field equations in flat space for free fields. The problems appear when one considers interactions of these fields. The most obvious interaction is the gravitational interaction. An immediately way of introducing the latter is to replace ordinary derivatives with covariant ones in order to maintain general covariance. However, in this case gauge invariance is lost as we need to commute derivatives in the field equations [3]. In fact only in flat Minkowski spacetime derivatives commute and gauge invariance is possible. Indeed, for massless fields of spin $s > \frac{1}{2}$, the field equations for bosons take schematically the form $D_\mu F^{\mu\mu_2\ldots} = 0$, where $F^{\mu\mu_2\ldots}$ is the antisymmetric field strength, a generalization of the Maxwell $F^{\mu\nu}$ tensor [16]. Then the Bianchi identity $D_\mu D_\nu F^{\mu\nu\mu_3\ldots} = 0$ leads, by the non-commutativity of the covariant derivatives, to local constraints of the form $W_{\mu\nu\kappa\lambda} F^{\mu\nu\ldots} = 0$. As these constraints involve the Weyl tensor, i.e, the part of the Riemann tensor which is not specified by the gravitational field equations, minimal coupling of such field to gravity are not in general consistent. An exception is for spin $s = 1, 2$, which involve only Ricci and curvature scalar terms. The same happens with half-integer HS fields. This means that HS fields minimally coupled to gravity have acausal propagation in curved spacetimes and cannot consistently be defined. As a general rule, gauge invariance and general covariance cannot be simultaneously imposed, indicating the inconsistency of a minimal couplings of HS fields with spin $s > 2$ to gravity. This "no-go theorem" can however be circumvented on backgrounds with vanishing Weyl tensor, i.e., on conformally flat space-times, such as de Sitter (dS) and anti-de Sitter (AdS) spacetimes [18]. Indeed, soon after the results of [1],[2], propagation of HS fields on (A)dS have been discussed in [8]. In particular, by gauging an infinite-dimensional generalization of the target space Lorentz algebra, consistent interactions of HS fields has been introduced [6],[7]. However such consistent interactions do not have a flat spacetime limit as they are based on generally covariant curvature expansion on (A)dS spacetime with expansion parameter proportional to the (A)dS length.

Here we will discuss HS fields living not in the whole of AdS spacetime, but rather in a part of it. A particular example of such spacetimes, once boundaries are introduced, is the Randall-Sundrum one [19], which has extensively be studied as an alternative to compactification and in connection with the hierarchy problem [20]. The aim is twofold. Firstly, to study localization properties of HS fields in the 4D boundary of the anti-de Sitter space and secondly, to relate bulk fields to HS operators in the dual boundary theory. For this, we will study the reduced 4D theory for HS fields. In this case, HS fields may also have consistent gravitational interactions on flat spacetime (brane), although the whole tower of massive KK modes of the bulk fields are needed. Moreover, we find cubic couplings of the HS fields to gravity by introducing non-minimal terms in the action, as well as possible couplings of the HS fields to matter living in the brane.

In the RS2 background the holographic boundary theory is a strongly coupled CFT defined with a cutoff and coupled to 4D gravity, whereas in RS1, the boundary theory is a badly broken CFT in the IR [21]. In this framework, we will examine the holographic description in RS backgrounds as well as in the AdS spacetime.

In the next section 2, we discuss briefly the geometric setup and the boundary conditions needed. In section 3 and 4, we solve the HS bulk equations and we find the 4D spectrum for bosons and fermions, respectively. In sections 5, we present a gauge invariant on-shell action with cubic interactions and couplings of the HS fields to matter on the brane. In section 6, we discuss

the holography in AdS_5 for HS bosons and in RS for HS fermions. Finally, in section 7, we conclude by summarizing our results.

2. HIGHER SPINS IN A BOX: AdS_5 WITH BOUNDARIES

We will mainly consider here five-dimensional anti-de Sitter bulk spacetime AdS_5 with four-dimensional boundaries. In this case and in order to set up the notation, let us recall that AdS is a maximally symmetric spacetime. Its Riemann tensor is given in terms of its metric as

$$R_{\alpha\beta\mu\nu} = -\frac{\Lambda}{4}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}) \quad (2.1)$$

where $-\Lambda < 0$ is the five-dimensional cosmological constant. In Gaussian-normal coordinates, the metric takes the form

$$ds^2 = e^{-2\sigma} \eta_{ab} dx^a dx^b + dy^2, \quad (2.2)$$

where $a, b, \dots = 0, \dots, 3$, $y = x_5$ and $\sigma = \sigma(y)$. In this coordinates, eq.(2.1) gives

$$\sigma' = \pm\sqrt{\Lambda}/2, \quad \sigma'' = 0, \quad (2.3)$$

where a prime denotes derivative with respect to the normal coordinate y , i.e., $(') = \partial/\partial y$. For a smooth AdS_5 spacetime, eq.(2.3) is solved for $\sigma = \sqrt{\Lambda}/2 y$ with $-\infty < y < \infty$. A Randall-Sundrum (RS) background [19] now, is an AdS spacetime invariant under $y \rightarrow -y$. As a result, it may be viewed as a Z_2 orbifolds of AdS , and thus, only its positive section $y \geq 0$ may be considered. In addition, in the first model (RS1) of [19], there exists an “end of the world” at $y = \pi R$ so that $0 \leq y \leq \pi R$. By taking the limit $R \rightarrow \infty$, the second boundary at $y = \pi R$ is removed to infinity and one ends up with the second model (RS2) of [19]. Hence, in the RS1 background we have $0 \leq y \leq \pi R$, while $0 \leq y < \infty$ in RS2. On the other hand, for both RS1 and RS2 models, the positive root of $\sigma' = \sqrt{\Lambda}/2$ for $y > 0$ and the negative one for $y < 0$ may be used, so that we have

$$\sigma = 2a|y|, \quad a = \sqrt{\Lambda}/4.$$

However, with this form of the wrap factor, the second derivative of σ , which enters in the curvature tensors does not vanish but rather gives δ -function contributions to both Riemann and Ricci tensors. These contributions may be cancelled by putting branes of appropriate fine-tuned tensions at the fixed points of the Z_2 orbifold. The branes are 4D flat Minkowski spacetimes and they are the boundaries of the bulk AdS background. The boundary at $y = 0$ is the UV brane whereas the brane at $y = \pi R$ is the IR one. In RS1 our Universe is on the IR brane. In this way a possible solution of the hierarchy problem has been suggested. However, here a negative Newtonian constant appear. In the second model RS2 of [19] instead (where $R \rightarrow \infty$), our Universe is on the UV brane as the IR one has been removed to infinity. This model is considered as a valid alternative to compactification.

The fields living in the bulk are as if they were propagating in AdS but now they will, in addition, experience two boundaries at $y = 0$ and $y = \pi R$ in RS1, or just one boundary in RS2. Our task is to discuss the localization problem and the effective theory living on these 4D boundaries for integer spin fields with spin $s \geq 1$ and semi-integer spinors with spin $s \geq 3/2$. Spin 0, 1 and $1/2, 3/2$ are particular cases and have already been discussed [23],[26].

In curved spacetime, one has to modify the definition of the spacetime covariant derivative in order to maintain a local Lorentz invariance of a semi-integer spin field. This is achieved by introducing the covariant derivative

$$D_\mu = \nabla_\mu + \Gamma_\mu , \quad (2.4)$$

where ∇_μ is the spacetime covariant derivative and the spin connection is defined as

$$\Gamma_\mu = \frac{1}{2} \Sigma^{ab} e_a^\nu \nabla_\mu e_{b\nu} . \quad (2.5)$$

Here Σ^{ab} are the local generators of Lorentz symmetry and e_a^μ is the n-bein. For an AdS_{p+1} spacetimes with cosmological constant $-\Lambda$, ($\Lambda > 0$), one may introduce the $SO(2, p)$ -covariant derivative

$$\bar{\nabla}_M = D_M + \left(\frac{\Lambda}{4p} \right)^{1/2} \gamma_M , \quad M = 0, \dots, p , \quad (2.6)$$

where γ_M are the $(p+1)$ -dimensional gamma matrices. In particular, the $SO(2, 4)$ -covariant derivative for AdS_5 is

$$\bar{\nabla}_\mu = D_\mu + a \gamma_\mu , \quad (2.7)$$

and, in Gaussian-normal coordinates, the spin connection in AdS_5 is

$$\Gamma_a = \frac{1}{2} \gamma_5 \gamma_a \sigma' , \quad \Gamma_5 = 0 . \quad (2.8)$$

Defining as usual $\gamma^{\mu\nu} = \frac{1}{2} [\gamma^\mu, \gamma^\nu]$ and using the relation $[\gamma_\nu, D_\nu] = 0$, a straightforward calculation explicitly shows that for a fermion Ψ

$$[D_\mu, D_\nu] \Psi = \frac{1}{4} R_{\mu\nu\alpha\beta} \gamma^{\alpha\beta} \Psi . \quad (2.9)$$

For a general tensor-spinor $\epsilon_{\alpha_1 \dots \alpha_s}$ of rank s , the $SO(2, 4)$ -covariant derivative satisfies

$$[\bar{\nabla}_\mu, \bar{\nabla}_\nu] \epsilon_{\alpha_1 \dots \alpha_s} = [D_\mu, D_\nu] \epsilon_{\alpha_1 \dots \alpha_s} + 2a^2 \gamma_{\mu\nu} \epsilon_{\alpha_1 \dots \alpha_s} . \quad (2.10)$$

A central issue when boundaries are present, as in the RS background, is the boundary condition problem. The variation of the action introduces boundary terms, which appear in the integration by parts during the variational process. These boundary terms must vanish independently from the bulk terms, which provide the equations of motion, and introduce appropriate boundary conditions. For fermionic fields for example, the action is of the form

$$S = \frac{1}{2} \int d^D x \sqrt{-g} \Psi_{\alpha_1 \dots \alpha_{s-1/2}} \gamma^\beta \bar{\nabla}_\beta \Psi^{\alpha_1 \dots \alpha_{s-1/2}} + \dots . \quad (2.11)$$

In the presence of boundaries, the variation of the above action provides boundary terms, which should vanish

$$(\delta \Psi^L \cdot \Psi^R - \delta \Psi^R \cdot \Psi^L) \Big|_{0, \pi R} = 0 . \quad (2.12)$$

We have denoted by a dot the inner product, i.e., $(\Psi \cdot \Psi = \Psi_{\alpha_1 \dots \alpha_{s-1/2}} \Psi^{\alpha_1 \dots \alpha_{s-1/2}})$. As we are interested in the Z_2 symmetry $y \rightarrow -y$, it is easy to see that the action S is Z_2 symmetric if $\Psi(-y) = \pm \gamma^5 \Psi(y)$, or

$$\Psi^L(-y) = \pm \Psi^L(y) ,$$

$$\Psi^R(-y) = \mp \Psi^R(y) . \quad (2.13)$$

Without loss of generality we can choose the positive root. This means that the right-handed field will in general have a “kink” profile passing throughout $y = 0$. Considering therefore only the positive domain $y > 0$, one can use the following boundary conditions

$$\begin{aligned} \text{(i)} \quad & \Psi^L(\pi R) = \Psi^R(\pi R), \quad \Psi^L(0^+) = \Psi^R(0^+), \\ \text{(ii)} \quad & \Psi^L(0^+) = \Psi^L(\pi R) = 0, \\ \text{(iii)} \quad & \Psi^R(0^+) = \Psi^R(\pi R) = 0, \end{aligned}$$

However the boundary conditions (ii) and (iii) can be modified allowing a non-zero mass at the UV boundary. This mass term at the boundary will be crucial for the holographic interpretation as we shall see later.

With a similar procedure, we can consider very schematically a bosonic field with action

$$S = \frac{1}{2} \int d^D x \sqrt{-g} \nabla_\mu \Phi_{\alpha_1 \dots \alpha_s} \nabla^\mu \Phi^{\alpha_1 \dots \alpha_s} + \dots . \quad (2.14)$$

Without the Z_2 symmetry, the variational principle, in gaussian-normal coordinates, is well defined if

$$(\delta \Phi \cdot n^a \partial_a \Phi) \Big|_{0, \pi R} = 0 . \quad (2.15)$$

However as the spacetime is Z_2 symmetric, the bulk field variation has a term like

$$\delta \Phi_{\alpha_1 \dots \alpha_s} \nabla_\mu \nabla^\mu \Phi^{\alpha_1 \dots \alpha_s} , \quad (2.16)$$

which in fact contain a boundary term on the fix points of the spacetime. This happens because the second derivative of the metric is distributional on the fix points. Such second derivative is coming from terms containing $n^a \partial_a \Gamma$, where we schematically use Γ to mean the discontinuous part of the Christoffel symbols. With that a boundary term like

$$(s-1) \delta \Phi \cdot \Phi n^a \partial_a \Gamma , \quad (2.17)$$

arises. Then, in gaussian-normal coordinate on an AdS background, we obtain the following two possible boundary conditions for a bosonic field Φ of any spin

a. Neumann

$$\Phi'(y) + (s-1) \sigma' \Phi(y) \Big|_{0, \pi R} = 0 , \quad (2.18)$$

b. Dirichlet

$$\Phi(y) \Big|_{0, \pi R} = 0 . \quad (2.19)$$

3. INTEGER SPINS IN BRANEWORLD

A generic field of spin s is described by a totally symmetric rank- s tensor $\phi_{\mu_1 \mu_2 \dots \mu_s}$. As we shall discuss briefly at the end of this section and in Section 6, its field equations on a smooth AdS_{p+1} spacetime may be written as

$$\nabla^2 \phi_{\mu_1 \mu_2 \dots \mu_s} - M^2 \phi_{\mu_1 \mu_2 \dots \mu_s} = 0 , \quad (3.1)$$

where the covariant derivatives are with respect to the AdS background. It can in fact be proven that eq.(3.1) is invariant under the gauge transformation

$$\delta\phi_{\mu_1\mu_2\ldots\mu_s} = \nabla_{(\mu_1}\xi_{\mu_2\mu_3\ldots\mu_s)} \quad (3.2)$$

only for the particular value

$$M^2 = \frac{\Lambda}{p} (s^2 - (5-p)s - 2p + 4),$$

provided $\phi_{\mu_1\mu_2\ldots\mu_s}$ satisfies the gauge condition

$$\nabla_\nu \phi^\nu_{\mu_2\mu_3\ldots\mu_s} - \frac{1}{2} \nabla_{(\mu_2} \phi^\nu_{\mu_3\ldots\mu_s)} \nu = 0. \quad (3.3)$$

This may easily be verified by taking into account that

$$\nabla^2 \xi_{\mu_2\mu_3\ldots\mu_s} = \frac{\Lambda}{p} (p + s - 2)(s - 1) \xi_{\mu_2\mu_3\ldots\mu_s}, \quad (3.4)$$

which follows from eqs.(3.2,3.3). In particular, for AdS_5 ($p = 4$), gauge invariance is achieved for

$$M^2 = 4a^2(s^2 - s - 4). \quad (3.5)$$

However, in a RS background, the HS field equations eq.(3.1) are not invariant under the gauge transformation eq.(3.2). The reason is that in this case there are δ -function contributions coming from the Riemann and Ricci tensors. These contributions spoil gauge invariance, which can be restored, nevertheless, by adding appropriate terms to the field equation (3.1). For example, the gauge variation of (3.1) for the spin s component of the reduced field $\phi_{m_1\ldots m_s}$, (Latin indices take the values $m, n = 0, \ldots, 3$), turns out to be

$$\delta(\nabla^2 \phi_{m_1\ldots m_s} - M^2 \phi_{m_1\ldots m_s}) = -4a(s-2)\delta(y)\delta\phi_{m_1\ldots m_s}, \quad (3.6)$$

so that the gauge invariant field equations for HS fields in RS background for $\phi_{m_1\ldots m_s}$ is

$$\nabla^2 \phi_{m_1\ldots m_s} - M^2 \phi_{m_1\ldots m_s} + 4a(s-2)\delta(y)\phi_{m_1\ldots m_s} = 0. \quad (3.7)$$

It turns out, after explicitly calculating (3.7), that the above field equations, in the gauge $\phi_{5\mu_2\ldots\mu_s} = 0$, are written as

$$e^{2\sigma} \partial_m \partial^m \phi_{m_1\ldots m_s} + \phi''_{m_1\ldots m_s} + 2(2-s)\sigma' \phi'_{m_1\ldots m_s} + ((s(s-1)-4s)\sigma'^2 - M^2) \phi_{m_1\ldots m_s} = 0 \quad (3.8)$$

supplemented with the boundary condition

$$\phi'_{m_1\ldots m_s} + 4a(s-1)\phi_{m_1\ldots m_s} \Big|_{0,\pi R} = 0. \quad (3.9)$$

We may write, denoting collectively indices by dots,

$$\phi_{\ldots}(x, y) = \sum_n f_n(y) \varphi_{\ldots}^{(n)}(x) \quad (3.10)$$

where $\varphi_{\ldots}(x)$ is an ordinary massive spin- s field in Minkowski spacetime

$$\partial_m \partial^m \varphi_{\ldots}^{(n)}(x) = m_n^2 \varphi_{\ldots}^{(n)}(x). \quad (3.11)$$

Then $f_n(y)$ satisfies the equation

$$e^{2\sigma} m_n^2 f_n + f_n'' - 4a(2-s)f_n' + 16a^2(1-s)f_n = 0, \quad (3.12)$$

with boundary conditions

$$f_n' + 4a(s-1)f_n|_{0,\pi R} = 0. \quad (3.13)$$

The solution of eq.(3.12) is

$$f_n = \frac{e^{2a(2-s)|y|}}{N_n} \left(J_s\left(\frac{m_n e^{2a|y|}}{2a}\right) + b_\nu(m_n) Y_s\left(\frac{m_n e^{2a|y|}}{2a}\right) \right), \quad (3.14)$$

where ν is the order of the Bessel's functions appearing in the solution.

For completeness, it should be noted that the corresponding solution in a $(p+1)$ -dimensional space AdS_{p+1} is

$$f_n = \frac{e^{a(p-2s)|y|}}{N_n} \left(J_{2-s-\frac{p}{2}}\left(\frac{m_n e^{2a|y|}}{2a}\right) + b_\nu(m_n) Y_{2-s-\frac{p}{2}}\left(\frac{m_n e^{2a|y|}}{2a}\right) \right), \quad (3.15)$$

which clearly reduces to eq.(3.14) for $p=4$.

For canonically normalized 4D fields $\varphi_{...}^{(n)}$, the normalization of f_n in RS1 should be

$$\int_0^{\pi R} dy e^{4a(s-1)y} f_n^* f_m = \delta_{mn}. \quad (3.16)$$

Therefore, the parameter N_n in eqs.(3.14) are

$$N_n^2 = \int_0^{\pi R} dy e^{2\sigma} \left[J_s\left(\frac{m_n e^{2a|y|}}{2a}\right) + b_\nu Y_s\left(\frac{m_n e^{2a|y|}}{2a}\right) \right]^2, \quad (3.17)$$

and by employing the boundary conditions (3.13) we get the relations

$$\begin{aligned} b_\nu(m_n) &= -\frac{sJ_\nu\left(\frac{m_n}{2a}\right) + \frac{m_n}{2a}J_\nu'\left(\frac{m_n}{2a}\right)}{sY_\nu\left(\frac{m_n}{2a}\right) + \frac{m_n}{2a}Y_\nu'\left(\frac{m_n}{2a}\right)}, \\ b_\nu(m_n) &= b_\nu(m_n e^{2a\pi R}). \end{aligned} \quad (3.18)$$

Accordingly, for the $(p+1)$ -dimensional space AdS_{p+1} , applying (3.13) to the general solution (3.15), we get

$$\begin{aligned} b_\nu(m_n) &= -\frac{(2s-p+4)J_\nu\left(\frac{m_n}{2a}\right) + \frac{m_n}{a}J_\nu'\left(\frac{m_n}{2a}\right)}{(2s-p+4)Y_\nu\left(\frac{m_n}{2a}\right) + \frac{m_n}{a}Y_\nu'\left(\frac{m_n}{2a}\right)}, \\ b_\nu(m_n) &= b_\nu(m_n e^{2a\pi R}). \end{aligned} \quad (3.19)$$

The conditions (3.18) specify both b_n and the mass spectrum m_n . There is also a zero mode corresponding to $m_n = 0$ in eq.(3.12). The (normalized) zero mode solution is easily found to be

$$f_0 = \frac{1}{\pi R} e^{-4a(s-1)|y|}. \quad (3.20)$$

It should be noted that in order the zero mode to exists, the singular term in eq.(3.7) is necessary. This term modifies the boundary conditions appropriately and allows the existence of the zero

mode f_0 . In particular, if we denote by S_{bulk} the *effective* bulk action in an AdS background which describes the dynamics of the $\phi_{m_1\dots m_s}$ field, the term which accounts for the singular term in (3.7) is

$$S = S_{\text{bulk}} + 4a(s-2) \int d^5x \sqrt{-g_{\text{ind}}} \delta(y) \frac{1}{2} \phi_{m_1\dots m_s} \phi^{m_1\dots m_s}. \quad (3.21)$$

This extra singular term corresponds to an irrelevant deformation of the boundary CFT and it has also been proposed in the AdS/CFT context in [28].

More specifically, the bulk gauge invariant action in (3.21), is identical to the action of a bosonic HS field in an exact AdS_5 background, which turns out to be [8], [29]

$$\begin{aligned} S_{\text{bulk}} = & - \int d^5x \sqrt{-g} \left(\frac{1}{2} \nabla_\mu \Phi_{\alpha_1\dots\alpha_s} \nabla^\mu \Phi^{\alpha_1\dots\alpha_s} - \frac{1}{2} s \nabla_\mu \Phi^\mu_{\alpha_2\dots\alpha_s} \nabla_\nu \Phi^{\nu\alpha_2\dots\alpha_s} \right. \\ & + \frac{1}{2} s(s-1) \nabla_\mu \Phi^\nu_{\nu\alpha_3\dots\alpha_s} \nabla^\kappa \Phi_{\kappa\alpha_3\dots\alpha_s} - \frac{1}{4} s(s-1) \nabla_\mu \Phi^\kappa_{\kappa\alpha_2\dots\alpha_s} \nabla^\mu \Phi^\lambda_{\lambda\alpha_2\dots\alpha_s} \\ & - \frac{1}{8} s(s-1)(s-2) \nabla_\mu \Phi^{\mu\kappa}_{\kappa\alpha_4\dots\alpha_s} \nabla^\nu \Phi_{\nu\lambda}^{\lambda\alpha_4\dots\alpha_s} \\ & \left. + 2a^2 (s^2 - s - 4) \Phi_{\alpha_1\dots\alpha_s} \Phi^{\alpha_1\dots\alpha_s} - a^2 s(s-1)(s^2 + s - 4) \Phi^\mu_{\mu\alpha_2\dots\alpha_s} \Phi^\nu_{\nu\alpha_2\dots\alpha_s} \right). \end{aligned} \quad (3.22)$$

The derivatives are covariant derivatives with respect to the AdS space. This action is invariant under the transformation (3.2) in an exact AdS_5 background, as we shall discuss in Section 6. As we have already mentioned, one need to commute covariant derivatives in order to prove gauge invariance. These commutations produce Riemann and Ricci tensors, which in the AdS case are proportional to the metric and can completely be cancelled by the last two terms in (3.22). However, in the RS case, there are additional terms which are not cancelled and emerge from the singular part of the Riemann and Ricci tensors. Denoting these parts by ΔR , we have for example

$$\Delta R^5_{m5n} = -4a\delta(y)e^{-2\sigma}\eta_{mn}, \quad \Delta R_{mn} = -4a\delta(y)e^{-2\sigma}\eta_{mn}, \quad \Delta R_{55} = 16a\delta(y). \quad (3.23)$$

As a result, in the gauge variation of (3.22), there are uncanceled terms proportional to the singular ΔR . Nevertheless, these contributions can still be cancelled by adding appropriate terms to (3.22). One may prove that indeed, the action

$$S' = S + S_\delta \quad (3.24)$$

is gauge invariant in a RS background for

$$\begin{aligned} S_\delta = & \frac{1}{2} \int d^5x \sqrt{-g} \left(\frac{1}{2} s^2 (s-1)(s-2) \Phi^{\kappa\mu\alpha_4\dots\alpha_s}_{\kappa} \Delta R^\nu_{\mu} \Phi^\lambda_{\alpha_4\dots\alpha_s\nu\lambda} \right. \\ & + s(s-2) \Phi^{\mu\alpha_2\dots\alpha_s} \Delta R^\nu_{\mu} \Phi_{\alpha_2\dots\alpha_s\nu} \\ & - s(s-2)(s^2 - 5s - 4) \Phi^{\alpha_1\dots\alpha_s} \Delta R_{\alpha_1}^{\mu\nu} \Phi_{\alpha_3\dots\alpha_s\mu\nu} \\ & \left. - \frac{1}{2} s^2 (s-1)^2 (s^2 - 5s - 4) \Phi^{\kappa\mu\alpha_4\dots\alpha_s}_{\kappa} \Delta R_{\alpha_1}^{\mu\nu} \Phi^\lambda_{\alpha_5\dots\alpha_s\mu\nu\lambda} \right). \end{aligned} \quad (3.25)$$

Obviously, S_δ is a boundary term as it is proportional to $\delta(y)$. Then, the transverse traceless part of the HS satisfies eq.(3.1) and S_δ reduces to the singular part of (3.21) as expected.

4. HALF-INTEGER SPINS IN BRANEWORLD

We will now study fermionic fields of half-integer spin s propagating in the bulk of AdS spacetimes. Such fields are described by totally symmetric tensor-spinors of rank $s - \frac{1}{2}$, $\Psi_{\alpha_1 \dots \alpha_{s-1/2}}$, and their dynamics is governed by the equation

$$\gamma^\rho \bar{\nabla}_\rho \Psi_{\alpha_1 \dots \alpha_{s-1/2}} - \gamma^\rho \bar{\nabla}_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_{s-1/2})\rho} + \beta \Psi_{\alpha_1 \dots \alpha_{s-1/2}} = 0 . \quad (4.1)$$

It can straightforward be proven that (4.1) in AdS_{p+1} is invariant under the gauge transformation

$$\delta \Psi_{\alpha_1 \dots \alpha_{s-1/2}} = \bar{\nabla}_{(\alpha_1} \epsilon_{\alpha_2 \dots \alpha_{s-1/2})} , \quad (4.2)$$

when the gauge parameter satisfies the constraint $\gamma^\rho \epsilon_{\alpha_1 \dots \rho \dots \alpha_{s-3/2}} = 0$, for the particular value

$$\beta = (2s - 3) \sqrt{\frac{\Lambda}{p}} . \quad (4.3)$$

We proceed to solve eq.(4.1) in slices of AdS_5 spacetime, where β is now given according to (4.3) by

$$\beta = 2a(2s - 3) . \quad (4.4)$$

Note that the $s = 1/2, 3/2$ cases have been studied in [22],[23],[24]. By using the gauge condition $\gamma^\rho \Psi_{\rho \dots} = 0$, eq.(4.1) simplifies to

$$\gamma^\rho D_\rho \Psi_{\alpha_1 \dots \alpha_{s-1/2}} + 2as \Psi_{\alpha_1 \dots \alpha_{s-1/2}} = 0 . \quad (4.5)$$

We may impose the conditions

$$\Psi_{\alpha_1 \dots 5 \dots \alpha_{s-1/2}} = 0 = \Psi_{\alpha_1 \dots \mu \dots \alpha_{s-1/2}} , \quad (4.6)$$

as they are consistent with the field equations (4.1). In the sequel, it is convenient to introduce the new fields $\Phi_{a_1 \dots a_{s-1/2}}$, defined by

$$\Phi_{a_1 \dots a_{s-1/2}} = e^{\sigma(s-5/2)} \Psi_{a_1 \dots a_{s-1/2}} , \quad a_1, a_2, \dots = 0, 1, 2, 3 . \quad (4.7)$$

Projecting in left/right (L/R) chirality modes, we obtain the following two coupled differential equations for these fields in AdS_5 spacetime

$$\begin{aligned} \gamma^c \partial_c \Phi_{a_1 \dots a_{s-1/2}}^R + \partial_5 \Phi_{a_1 \dots a_{s-1/2}}^L + 2as \Phi_{a_1 \dots a_{s-1/2}}^L &= 0 \\ \gamma^c \partial_c \Phi_{a_1 \dots a_{s-1/2}}^L - \partial_5 \Phi_{a_1 \dots a_{s-1/2}}^R + 2as \Phi_{a_1 \dots a_{s-1/2}}^R &= 0 . \end{aligned} \quad (4.8)$$

We can solve the above system by separation of variables

$$\Phi_{\alpha_1 \dots \alpha_{s-1/2}}^{L,R} = \sum_n f_{L,R}^{(n)}(y) \psi_{\alpha_1 \dots \alpha_{s-1/2}}^{(n)}(x^a) . \quad (4.9)$$

Recalling that $\gamma^a = e^\sigma \hat{\gamma}^a$ where $\hat{\gamma}^a$ are the gamma matrices in flat Minkowski spacetime, we consider the eigenvalue problem

$$\hat{\gamma}^a \partial_a \psi_{\alpha_1 \dots \alpha_{s-1/2}}^{(n)} = m_n \psi_{\alpha_1 \dots \alpha_{s-1/2}}^{(n)} , \quad (4.10)$$

which defines the 4D HS spectrum. The system of eqs.(4.8) reduce then to the first order differential equations

$$\begin{aligned} e^\sigma m_n f_R + f'_L + 2as f_L &= 0 \\ e^\sigma m_n f_L - f'_R + 2as f_R &= 0, \end{aligned} \quad (4.11)$$

which, in terms of the new variable $z = \frac{e^\sigma m_n}{2a}$, are written as

$$\begin{aligned} f_R + \partial_z f_L + \frac{s}{z} f_L &= 0 \\ f_L - \partial_z f_R + \frac{s}{z} f_R &= 0. \end{aligned} \quad (4.12)$$

The solution of the above equations is

$$\begin{aligned} f_L &= \frac{z^{1/2}}{N_n} \left(J_{-s-\frac{1}{2}}(z) + B_n(m_n) Y_{-s-\frac{1}{2}}(z) \right), \\ f_R &= \frac{z^{1/2}}{N_n} \left(J_{-s+\frac{1}{2}}(z) + B_n(m_n) Y_{-s+\frac{1}{2}}(z) \right), \end{aligned} \quad (4.13)$$

where J_ν and Y_ν are the Bessel functions. Moreover, the zero modes of the field, which correspond to $m_n = 0$ in (4.11), are

$$\begin{aligned} f_L &= f_L^0 e^{-s\sigma}, \\ f_R &= f_R^0 e^{s\sigma}. \end{aligned} \quad (4.14)$$

The boundary and normalization conditions fix the mass spectrum of the field and all the parameters of the solution (4.13). The normalization condition for the solutions is chosen such that

$$\int dy e^\sigma f_m f_n = \delta_{mn}. \quad (4.15)$$

This is equivalent to the requirement of a canonically normalized kinetic term for the 4D reduced HS fields $\psi_{\alpha_1 \dots \alpha_{s-1/2}}^{(n)}$. It can explicitly be written as

$$N_n^2 = \int_0^{\pi R} dy e^{2\sigma} \left[J_\nu \left(\frac{m_n e^{2a|y|}}{2a} \right) + b_\nu Y_\nu \left(\frac{m_n e^{2a|y|}}{2a} \right) \right]^2 \quad (4.16)$$

where $\nu = -s - \frac{1}{2}$ and $\nu = -s + \frac{1}{2}$ for the left and right modes, respectively. Moreover, the boundary condition (i) in eq.(2.14) are written in the present case as

$$f_R(0^+) = f_L(0^+), \quad f_R(\pi R) = f_L(\pi R), \quad (4.17)$$

and specifies the parameter B and the masses spectrum. Indeed, we get

$$\begin{aligned} B(m_n) &= \frac{J_{-s+\frac{1}{2}}\left(\frac{m_n}{2a}\right) - J_{-s-\frac{1}{2}}\left(\frac{m_n}{2a}\right)}{Y_{-s-\frac{1}{2}}\left(\frac{m_n}{2a}\right) - Y_{-s+\frac{1}{2}}\left(\frac{m_n}{2a}\right)}, \\ B(m_n) &= B(m_n e^{2a\pi R}). \end{aligned} \quad (4.18)$$

There is no analytical solution for the mass spectrum, but instead we have plotted the function $f_R(\pi R) - f_L(\pi R)$ in fig.(1) and fig.(2). The set of zeros correspond to the mass spectrum. We

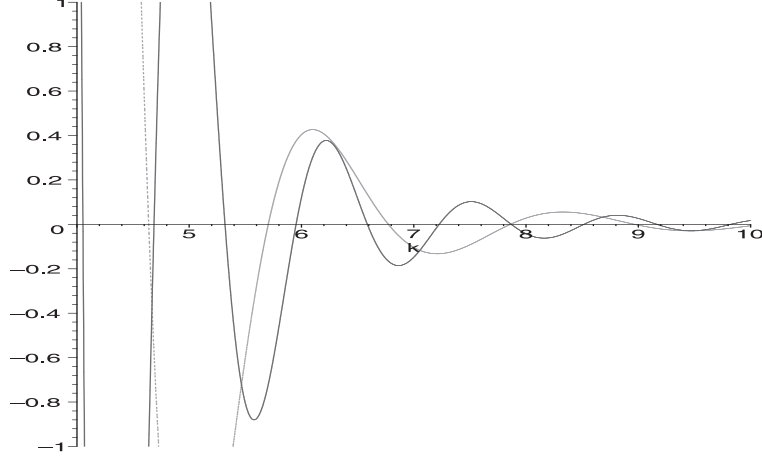


FIG. 1: Zeros of the function $10^{-2}N_n(f_L - f_R)$ for the simple case of $s = 5/2$ with boundary conditions (i). In the horizontal axis we used the variable $k = m/a$. The dashed curve is for $R = \ln(5)/2\pi a$. The solid one is for $R = \ln(3)/2\pi a$. We note that the zeros tend to a continuum for increasing R .

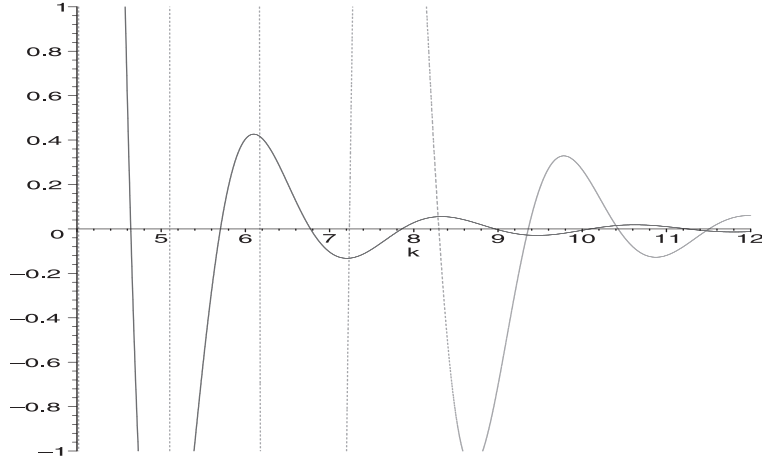


FIG. 2: Zeros of the function $10^{-2}N_n(f_L - f_R)$ for $R = \ln(3)/2\pi a$ and boundary conditions (i). Solid line $s = 5/2$, dashed $s = 7/2$. We note that increasing the spin means to shift the mass spectrum further.

note that there exists an infinite tower of massive states for finite R . For R increasing the modes become closer one to each other until the limit of a continuum spectrum when $R \rightarrow \infty$. Moreover we note that the massless modes (4.14) do not satisfy the boundary conditions and thus it is not in the physical spectrum. As a result the spectrum corresponding to the boundary condition (i), which mixes left and right modes, consists of a tower of massive modes with no massless field.

However, the situation is different for the boundary conditions (ii) and (iii). For example, the boundary conditions (iii) give

$$B(m_n) = -\frac{J_{-s+\frac{1}{2}}(\frac{m_n}{2a})}{Y_{-s+\frac{1}{2}}(\frac{m_n}{2a})},$$

$$B(m_n) = B(m_n e^{2a\pi R}), \quad (4.19)$$

from where, the mass spectrum and the constants B are specified. In this case however, there is

the zero mode

$$f_L = f_L^0 e^{-s\sigma}, \quad f_R = 0, \quad (4.20)$$

localized at the $y = 0$ boundary. For the boundary condition (ii), left and right modes are interchanged and the zero mode is a right handed one localized at the $y = \pi R$ boundary. However, in this case, by moving the boundary to infinity, the right-handed zero mode blows up, and thus disappears from the physical spectrum in RS2 due to its non-normalizability.

5. CONSISTENT GRAVITATIONAL COUPLINGS

We have already noted in the introduction that a free HS theory can consistently be defined in Minkowski spacetime. We have also stressed that problems appear when minimal interactions are introduced. For example, if additional gauging, as proposed in [5],[6],[7], are not allowed, gauge breaking terms proportional to the Riemann tensor emerge. These terms are non zero even for on shell graviton and therefore tree-level unitarity is lost. The situation is different for massive HS fields. In this case, without introducing additional gauging, in order to cancel the gauge breaking terms of the massless theory, a non-minimal interaction like $\frac{1}{m}\Phi_{\alpha\beta\dots}\mathcal{R}^{\alpha\mu\nu\beta}\Phi_{\mu\nu\dots}$ has been proposed [32]. This interaction cancels hard gauge-breaking terms, i.e., terms that do not vanish at the massless limit, although gauge invariance is still softly broken due to an explicit mass term. With the addition of the above non-minimal interaction, the theory is lacking of any hard breaking terms at linearized level, which could violate tree-level unitarity. Hence, tree-level unitarity is restored up to the Planck scale [32],[33]. The price paid is the violation of the equivalence principle due to the introduction of the non-minimal interaction terms [32],[34]. Although such terms looks odd, experience from electromagnetic interactions, suggests that the physical requirement is the tree-level unitarity [30],[31] rather than minimal coupling. It is clear of course that the massless limit for the interactive theory is not defined.

In the case we are considering, we have seen in eqs.(3.1,4.1) that in order to have gauge invariance in AdS , a non-derivative term is needed. This is something like the mass term discussed above in the Minkowskian case. Similarly, when a gravitational perturbation is switched on, gauge breaking terms proportional to the Riemann tensor of the graviton appear again. As in [32], one can hope that an equivalent non-minimally coupled interaction may cancel the Riemann tensor contribution to the variation of the action under the gauge transformation for the HS field. Contrary to the four dimensional case in Minkowski background, in our case the non-derivative term does not break gauge invariance. Therefore the only cancellation of the hard terms restore the gauge invariance for the interacting theory, at least at linearized level. We will show below that it is actually possible to consistently make the interaction theory gauge invariant at linearized level. A different perspective has been introduced in [6] and it would be interesting to connect this approach with ours. However this is beyond the scope of this paper.

A. Non-minimally coupled lagrangian

For simplicity, in the following we concentrate on half integer spins, although the discussion can be generalized along the lines of [33] to include integer HS fields as well. For this, let us write the equation of motion (4.1) for a fermionic field $\Psi_{\alpha_1\dots\alpha_{s-1/2}}$ of spin s as

$$Q_{\alpha_1\dots\alpha_{s-1/2}} = 0, \quad (5.1)$$

where we have defined

$$Q_{\alpha_1 \dots \alpha_{s-1/2}} = \gamma^\rho \bar{\nabla}_\rho \Psi_{\alpha_1 \dots \alpha_{s-1/2}} - \gamma^\rho \bar{\nabla}_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_{s-1/2})\rho} + 2a(2s-3) \Psi_{\alpha_1 \dots \alpha_{s-1/2}} . \quad (5.2)$$

The action for this field is a generalization of the action in [2],[16] on Minkowski background and can be written as

$$S = \int d^5x \sqrt{-g} \left[-\frac{1}{2} \bar{\psi}_{\alpha_1 \dots \alpha_{s-1/2}} Q^{\alpha_1 \dots \alpha_{s-1/2}} + \frac{1}{4} \left(s - \frac{1}{2}\right) \bar{\psi}_{\mu\rho\alpha_3 \dots \alpha_{s-1/2}} \gamma^\rho \gamma_\sigma Q^{\sigma\mu\alpha_3 \dots \alpha_{s-1/2}} \right. \\ \left. + \frac{1}{8} \left(s - \frac{1}{2}\right) \left(s - \frac{3}{2}\right) \bar{\psi}^\mu_{\mu\alpha_3 \dots \alpha_{s-1/2}} Q^\nu_{\nu}{}^{\alpha_3 \dots \alpha_{s-1/2}} \right] \quad (5.3)$$

It should be noted that the field equations, which follows from this action, is not eq.(5.1) but rather

$$Q_{\alpha_1 \dots \alpha_{s-1/2}} - \frac{1}{2} \gamma_{(\alpha_1} \gamma^\rho Q_{\alpha_2 \dots \alpha_{s-1/2})\rho} - \frac{1}{2} g_{(\alpha_1 \alpha_2} Q^\mu_{\alpha_3 \dots \alpha_{s-1/2})\mu} = 0 . \quad (5.4)$$

However, contraction with the metric and with a γ -matrix gives

$$Q^{\mu\dots}{}_\mu = 0, \quad \gamma^\rho Q_{\rho\alpha_2 \dots \alpha_{s-1/2}} = 0, \quad (5.5)$$

so that (5.4) is actually equivalent to (5.1). There is no invariant action which leads directly to (5.1) without introducing auxiliary fields [16].

Concerning now the gauge invariance of (5.3), it can be checked by using the gauge condition $\gamma^\mu \epsilon_{\mu\dots} = 0$ and the Majorana-flip identity

$$\bar{\Psi}_{\alpha_1 \dots \alpha_{s-1/2}} \gamma^{\beta_1 \dots \beta_n} \xi_\lambda = (-)^n \bar{\xi}_\lambda \gamma^{\beta_1 \dots \beta_n} \Psi_{\alpha_1 \dots \alpha_{s-1/2}} . \quad (5.6)$$

Then, the variation of (5.3) after an integration by parts turns out to be

$$\delta S \sim \int d^5x \sqrt{-g} \bar{\epsilon}_{\alpha_2 \dots \alpha_{s-1/2}} \left(D_\mu Q^{\mu\alpha_2 \dots \alpha_{s-1/2}} - \gamma^{\rho\sigma} D_\rho Q_\sigma{}^{\alpha_2 \dots \alpha_{s-1/2}} \right. \\ \left. - D^{(\alpha_2} Q^{\alpha_3 \dots \alpha_{s-1/2})\mu}{}_\mu - 2s a \gamma_\mu Q^{\mu\alpha_2 \dots \alpha_{s-1/2}} \right) . \quad (5.7)$$

By an explicit computation one can show that the integrand in (5.7) actually vanishes for an AdS background (it is a contracted Bianchi-type identity). As a result, the HS theory described by (5.3) is gauge invariant on AdS .

Let us now consider possible coupling of the HS fermionic field with gravity in AdS background. In this case, after performing the gauge variation of (5.3), we will linearize in the gravitational field $h_{\mu\nu}$ and impose at the end the condition that both the HS field $\Psi_{\alpha_1 \dots \alpha_{s-1/2}}$ and the graviton are on-shell. Thus, we will employ the fermionic field equation, which in a covariant gauge can be written as

$$\gamma^\beta D_\beta \Psi_{\alpha_1 \dots \alpha_{s-1/2}} + 2as \Psi_{\alpha_1 \dots \alpha_{s-1/2}} = 0, \quad \gamma^\mu \Psi_{\mu\alpha_2 \dots \alpha_{s-1/2}} = 0, \quad (5.8)$$

as well as the graviton equation in the AdS background

$$R_{\mu\nu}(h) = -\Lambda h_{\mu\nu} . \quad (5.9)$$

By an explicit evaluation of (5.7), we then get that

$$\delta S = 2\left(s - \frac{3}{2}\right)^2 \int d^5x \sqrt{-g} \bar{\epsilon}_{\nu\alpha_3 \dots \alpha_{s-1/2}} \gamma^\mu W^\nu{}_{\beta\mu\lambda} \Psi^{\lambda\beta\alpha_3 \dots \alpha_{s-1/2}} , \quad (5.10)$$

where $W^\nu_{\alpha\mu\lambda}$ is the Weyl tensor. In the *AdS* case $W^\nu_{\alpha\mu\lambda} = 0$ and therefore S is gauge invariant when there is not coupling of the HS to gravity as we noted above. In fact, the only solutions of Einstein equation in vacuum (including a cosmological constant) with zero Weyl tensor are maximally symmetric. In this class there are only three possible spacetimes (*A*)*dS* or Minkowski. Therefore, as soon as a gravitational perturbation is switched on, the action (5.3) loses its gauge invariance.

In order to maintain gauge invariance of the HS action, we have to add a term, which will contain the Weyl tensor and it will be such that its gauge variation cancels the gauge breaking term (5.10). Let us therefore consider the interaction term

$$\Delta S_1 = -\frac{(2s-3)^2}{20a} \int d^5x \sqrt{-g} \bar{\Psi}_{\mu\nu\alpha_3\dots\alpha_{s-1/2}} \mathcal{W}^{\mu\rho\nu\sigma} \Psi_{\rho\sigma}{}^{\alpha_3\dots\alpha_{s-1/2}}, \quad (5.11)$$

where

$$\mathcal{W}^{\mu\rho\nu\sigma} = W^{\mu\rho\nu\sigma} - \frac{1}{2} W_{\alpha\beta}{}^{\rho\mu} \gamma^{\nu\sigma\alpha\beta}.$$

Another term which can be added and it is zero on an exact *AdS* background is

$$\Delta S_2 = \frac{(2s-3)^2}{160a^3} \int d^5x \sqrt{-g} \bar{\Psi}^\lambda{}_{\mu\nu\alpha_4\dots\alpha_{s-1/2}} \gamma_\lambda [(\alpha\bar{\Psi} + m)\bar{\nabla}_{\alpha_3} \mathcal{W}^{\mu\rho\nu\sigma}] \Psi_{\rho\sigma}{}^{\alpha_3\dots\alpha_{s-1/2}}, \quad (5.12)$$

where $m = 2a(s-5/2)$ is the coefficient of the non-kinetic term into the action, when gauge conditions are imposed and α is a free dimensionless parameter.

Clearly ΔS_2 can only be written for $s \geq 7/2$ and it does not contribute to the gravitational multipoles of the spin- s particle as it can be eliminated by gauge transformations of Ψ .

We will now show that the variation of the actions (5.11,5.12) exactly cancel the hard term on shell (which correspond to the first order in the perturbation theory) (5.10) up to a local redefinition of the fields. Since the identity for a Majorana spinors hold we can just concentrate on the variation of $\bar{\Psi}$

$$\delta\bar{\Psi}_{\mu\nu\alpha_3\dots\alpha_{s-1/2}} = D_{(\mu}\bar{\epsilon}_{\nu)A} - a\bar{\epsilon}_{A(\mu}\gamma_{\nu)} + (s-5/2) \left(D_{\alpha_3}\bar{\epsilon}_{\mu\nu\alpha_4\dots\alpha_{s-1/2}} - a\bar{\epsilon}_{\mu\nu\alpha_4\dots\alpha_{s-1/2}}\gamma_{\alpha_3} \right), \quad (5.13)$$

where we introduced the compact notation ($A = \alpha_3\dots\alpha_{s-1/2}$). It can easily be proven that the Weyl tensor satisfies

$$\nabla_\mu W_{\alpha\beta}{}^{\rho\nu}\gamma^{\mu\sigma\alpha\beta} = 0, \quad \nabla_\mu W^{\mu\rho\nu\sigma} = 0, \quad (5.14)$$

thanks to the Bianchi identities. Moreover using the gauge conditions, the cyclic identity and the fact that Ψ_{\dots} is a totally symmetric tensor, one can prove by direct computation that

$$W_{\alpha\beta}{}^{\rho[\mu}\gamma^{\nu]\sigma\alpha\beta} D_\mu \Psi_{\rho\sigma A} = 0. \quad (5.15)$$

Using these identities, after an integration by parts one gets

$$\delta\Delta S_1 = -\frac{(2s-3)^2}{10a} \int d^5x \sqrt{-g} \left\{ \bar{\epsilon}_{\nu A} W^{\mu\rho\nu\sigma} \left[D_{[\mu} \Psi_{\rho]\sigma}{}^A - \gamma^{\epsilon\lambda\mu\rho} D_\epsilon \Psi_{\sigma\lambda}{}^A \right] - (s-5/2) \{ \bar{\epsilon}_{\mu\nu\alpha_4\dots\alpha_{s-1/2}} \bar{\nabla}_{\alpha_3} (\mathcal{W}^{\mu\nu\rho\sigma} \Psi_{\rho\sigma}{}^{\alpha_3\dots\alpha_{s-1/2}}) \} \right\}. \quad (5.16)$$

Rearranging the γ matrices and considering the equation of motions $\gamma^\epsilon D_\epsilon \Psi_{\dots} = -2as\Psi_{\dots}$ one has

$$\delta\Delta S_1 = -2(s-3/2)^2 \int d^5x \sqrt{-g} \bar{\epsilon}_{\nu\alpha_3\dots\alpha_{s-1/2}} \gamma^\mu W^\nu{}_{\beta\mu\lambda} \Psi^{\lambda\beta\alpha_3\dots\alpha_{s-1/2}} +$$

$$+ \frac{(s-3/2)^2(s-5/2)}{5a} \int d^5x \sqrt{-g} \bar{\epsilon}_{\mu\nu\alpha_4\ldots\alpha_{s-1/2}} [\bar{\nabla}_{\alpha_3} \mathcal{W}^{\mu\rho\nu\sigma} \Psi_{\rho\sigma}^{\alpha_3\ldots\alpha_{s-1/2}}] . \quad (5.17)$$

The first line of (5.17) cancel the hard term (5.10). The total variation turns out then to be

$$\begin{aligned} \delta S + \delta\Delta S_1 + \delta\Delta S_2 &= -\frac{(2s-3)^3}{80a^3} \int d^5x \sqrt{-g} \bar{\epsilon}_{\mu\nu\alpha_4\ldots\alpha_{s-1/2}} \times \\ &\times [\alpha(\bar{\nabla}^2 - m^2) + \mathcal{M}(\bar{\nabla} + m)] \{ \bar{\nabla}_{\alpha_3} \mathcal{W}^{\mu\rho\nu\sigma} \Psi_{\rho\sigma}^{\alpha_3\ldots\alpha_{s-1/2}} \} , \end{aligned} \quad (5.18)$$

where $\mathcal{M} = 2a[s(2\alpha+1) - (4\alpha+5/2)]$. Under a local redefinition of Ψ the second line vanish on shell as it is proportional to the equation of motion for Ψ . As a result, the action $\delta S + \delta\Delta S_1 + \delta\Delta S_2$ is gauge invariant for on-shell interacting HS fields and gravitons.

B. Coupling to brane matter

The HS fields may also couple to matter living on the boundary branes. In order to find these couplings, we note that the interaction term (5.11) induces a boundary action when the variation of the metric vanishes at the boundary but not its orthogonal derivative. More explicitly, in gaussian normal coordinates, the boundary action appears whenever

$$\delta g_{ab} = 0 \quad \text{and} \quad \delta \partial_y g_{ab} = 2\delta K_{ab} \neq 0. \quad (5.19)$$

This is the case, for example, in the RS scenario. Then, when (5.19) is valid, by employing the gauge conditions $\Psi_{5\dots} = 0$ and the Majorana-flip identity, ΔS_1 in (5.11) reduces to

$$\Delta S_1 = -\frac{(s-3/2)^2}{5a} \int d^5x \sqrt{-g} \bar{\Psi}_{a_1 a_2 a_3 \dots a_{s-1/2}} \left[W^{a_1 b_1 a_2 b_2} - \frac{1}{2} W_{cd}^{b_1 a_1} \gamma^{a_2 b_2 cd} \right] \Psi_{b_1 b_2}^{a_3 \dots a_{s-1/2}}. \quad (5.20)$$

We recall that the Weyl tensor is expressed in terms of the curvature tensors as

$$W_{abcd} = R_{abcd} - \frac{1}{3} (g_{a[c} R_{d]b} - g_{b[c} R_{d]a}) + \frac{1}{12} R g_{a[c} g_{d]b}. \quad (5.21)$$

In Gaussian-normal coordinates, only the Ricci tensor and scalar contain a term proportional to the derivative of the extrinsic curvature [35]. In particular

$$R_{ab} = \partial_y K_{ab} + \dots . \quad (5.22)$$

By direct computation one can now show that the variation of the action (5.20) with respect to the metric is defined if and only if the following boundary term is added

$$S_b = -\frac{2(s-3/2)^2}{5a} \int_b d^4x \sqrt{-g_{ind}} \bar{\Psi}^a_{a_2 \dots a_{s-1/2}} \left[K_{ab} - \frac{1}{4} g_{ab} K \right] \Psi^{ba_2 \dots a_{s-1/2}}. \quad (5.23)$$

Note that, even if $\Psi_R(0^+) = -\Psi_R(0^-)$ we have that $S_b(0^+) = S_b(0^-)$ since $K(0^+) = -K(0^-)$.

In the case when $K_{ab}|_{0,\pi R} \propto g_{ab}|_{0,\pi R}$, the boundary action vanishes. This happens for example for the RS scenario. Here in fact only a boundary mass for the graviton is added. A second important thing to note is that only the massive modes which satisfy the boundary conditions (i) make the boundary action non vanishing. In fact, we have proved that the massless mode, if exists, is chiral and the boundary action mix right with left-handed modes.

If we now allow matter on the brane, the Israel-Darmois junction conditions relate the extrinsic curvature to the matter content on the brane as [36]

$$K_{ab}(0^+) = -\frac{1}{2}k_5^2 T_{ab}(0) + \dots, \quad (5.24)$$

where the dots indicate terms proportional to the induced metric, $k_5^{-2/3}$ is the five dimensional Planck mass and T_{ab} the energy momentum tensor of the boundary matter. Then an effective coupling between the HS-fields, treated as test fields, and matter on the brane appears, which may explicitly be written as

$$S_b = k_5^2 \frac{(s-3/2)^2}{5a} \int_b d^4x \sqrt{-g_{ind}} \bar{\Psi}^a_{a_2 \dots a_{s-1/2}} \left[T_{ab} - \frac{1}{4} g_{ab} T \right] \Psi^{ba_2 \dots a_{s-1/2}}. \quad (5.25)$$

Note that the boundary description we have presented here will break down when, using the RS fine tuning [36], $T_{ab} \sim a k_5^{-2} > 10 \text{TeV}^4$, where this limit is compatible with table-top tests of Newton's law (see [37] and references therein).

6. HOLOGRAPHY

In this section we will discuss HS fields in the AdS/CFT setup and their holographic interpretation. In particular, to make a contact with previous literature, we will explore bosonic HS fields in the standard AdS_5 case and fermionic HS fields in RS background.

A. Holography: bosons in AdS

Here, we will explicitly calculate the two-point function of higher-spin operators in the boundary CFT . The conformal dimension of HS operators, in the light-cone formalism, have been calculated in [38]. Here, using [39], the full two-point function of HS bosonic operators will be found including the tensorial structure. It should be noted that the $s = 0, 1$ cases have been evaluated initially in [39], whereas the $s = 2$ one in [40]. More references can be found in [41]. In general, the action for a bosonic HS fields in AdS_{p+1} is [29]

$$\begin{aligned} S = & -\int d^{p+1}x \sqrt{-g} \left(\frac{1}{2} \nabla_\mu \Phi_{\alpha_1 \dots \alpha_s} \nabla^\mu \Phi^{\alpha_1 \dots \alpha_s} - \frac{1}{2} s \nabla_\mu \Phi^\mu_{\alpha_2 \dots \alpha_s} \nabla_\nu \Phi^{\nu \alpha_2 \dots \alpha_s} \right. \\ & + \frac{1}{2} s(s-1) \nabla_\mu \Phi^\nu_{\nu \alpha_3 \dots \alpha_s} \nabla^\kappa \Phi_{\kappa}^{\mu \alpha_3 \dots \alpha_s} - \frac{1}{4} s(s-1) \nabla_\mu \Phi^\kappa_{\kappa \alpha_2 \dots \alpha_s} \nabla^\mu \Phi_{\lambda}^{\lambda \alpha_2 \dots \alpha_s} \\ & - \frac{1}{8} s(s-1)(s-2) \nabla_\mu \Phi^{\mu \kappa}_{\kappa \alpha_4 \dots \alpha_s} \nabla^\nu \Phi_{\nu \lambda}^{\lambda \alpha_4 \dots \alpha_s} \\ & \left. + \frac{\Lambda}{2p} (s^2 + (p-5)s - 2(p-2)) \Phi_{\alpha_1 \dots \alpha_s} \Phi^{\alpha_1 \dots \alpha_s} - \frac{\Lambda}{4p} s(s-1)(s^2 + (p-3)s - p) \Phi^\mu_{\mu \alpha_2 \dots \alpha_s} \Phi_{\nu}^{\nu \alpha_2 \dots \alpha_s} \right). \end{aligned} \quad (6.1)$$

where the derivatives are covariant derivatives in the AdS background. The field equations which follows from eq.(6.1) are

$$\begin{aligned} & \nabla^2 \Phi_{\alpha_1 \dots \alpha_s} - \nabla_{(\alpha_1} \nabla^\mu \Phi_{\alpha_2 \dots \alpha_s) \mu} + \frac{1}{2} \nabla_{(\alpha_1} \nabla_{\alpha_2} \Phi^\mu_{\alpha_3 \dots \alpha_s) \mu} + g_{(\alpha_1 \alpha_2} \nabla_\mu \nabla_\nu \Phi^{\mu \nu}_{\alpha_3 \dots \alpha_s)} - \nabla_{(\alpha_1} \nabla_\mu \Phi^\mu_{\alpha_2 \dots \alpha_s)} - \\ & - g_{(\alpha_1 \alpha_2} \nabla^2 \Phi^\mu_{\alpha_3 \dots \alpha_s) \mu} - \frac{1}{2} g_{(\alpha_1 \alpha_2} \nabla_{\alpha_3} \nabla^\mu \Phi^\nu_{\alpha_4 \dots \alpha_s) \mu \nu} - \frac{\Lambda}{p} (s^2 + (p-5)s - 2(p-2)) \Phi_{\alpha_1 \dots \alpha_s} \\ & + \frac{\Lambda}{p} (s^2 + (p-3)s - p) g_{(\alpha_1 \alpha_2} \Phi^\mu_{\alpha_3 \dots \alpha_s) \mu} = 0 \end{aligned} \quad (6.2)$$

The transverse traceless part of the HS fields $\nabla^\mu \Phi_{\mu\dots} = \Phi^\mu_{\mu\dots} = 0$ satisfy the free wave equation

$$\nabla^2 \Phi_{\alpha_1 \dots \alpha_s} - \frac{\Lambda}{p} (s^2 + (p-5)s - 2(p-2)) \Phi_{\alpha_1 \dots \alpha_s} = 0. \quad (6.3)$$

We remind again that the parenthesis in the indices denote symmetrization without combinatorial factors (i.e, $A_{(\mu} B_{\nu)} = A_\mu B_\nu + A_\nu B_\mu$). For later use, we note that when the equations of motions are obeyed, the action (6.1) turns out to be the total divergence

$$S = - \int d^{p+1} x \sqrt{-g} \nabla_\mu V^\mu, \quad (6.4)$$

where

$$\begin{aligned} V^\mu = & \frac{1}{2} \Phi^{\alpha_1 \dots \alpha_s} \nabla^\mu \Phi_{\alpha_1 \dots \alpha_s} - \frac{1}{2} s \Phi^{\mu \alpha_2 \dots \alpha_s} \nabla^\nu \Phi_{\nu \alpha_2 \dots \alpha_s} - \frac{1}{4} s(s-1) \Phi^{\mu \nu \alpha_3 \dots \alpha_s} \nabla_\nu \Phi^\lambda_{\lambda \alpha_2 \dots \alpha_s} \\ & - \frac{1}{4} s(s-1) \Phi^{\nu \nu \alpha_3 \dots \alpha_s} \nabla^\mu \Phi^\lambda_{\lambda \alpha_2 \dots \alpha_s} - \frac{1}{8} s(s-1)(s-2) \Phi^{\mu \kappa}_{\kappa \alpha_4 \dots \alpha_s} \nabla^\nu \Phi_{\nu \lambda}^{\lambda \alpha_4 \dots \alpha_s} \\ & + \frac{1}{4} s(s-1) \Phi^\nu_{\nu \alpha_3 \dots \alpha_s} \nabla^\lambda \Phi_\lambda^{\mu \alpha_3 \dots \alpha_s}. \end{aligned} \quad (6.5)$$

In the following we will restrict ourselves in the AdS_5 case so that $p = 4$, although the discussion may be kept more general. We will employ the conformally flat Poincaré coordinates for AdS_5 so that the metric is written as

$$ds^2 = \frac{1}{4a^2 x_0^2} (dx^a dx_a + dx_0^2), \quad (6.6)$$

moreover we will make use of the Euclidean signature.

There are two boundaries $x_0 = 0$ and $x_0 = \infty$. The $x_0 = 0$ boundary is the 4D Minkowski spacetime whereas the $x_0 = \infty$ one is actually a point as all the four dimensional points are shrank to zero. Thus, the boundary of AdS_5 is the 4D compactified Minkowski spacetime (Minkowski plus the point at infinity). To extract different four dimensional points from the boundary at infinity one can make an $SO(2,4)$ transformation that map the point $x_0 = \infty$ to $(x_0 = 0, x^a)$ and leave invariant the boundary at $x_0 = 0$. This transformation is an isometry for AdS_5 and correspond just to a conformal transformation on the Minkowskian boundary $x_0 = 0$. Our aim is to find the function Φ at the boundary in terms only of the boundary data $\phi_{a_1 \dots a_s}$ at $x_0 = 0$. We therefore look for a kind of propagator (Green function) for the field ϕ at the boundary. Since the point at infinity is mapped to the point at zero, it is much simpler to find a divergent solution of Φ at infinity and then map the point to zero. However, one has to be careful in taking the limit $x_0 = 0$ as some divergences may appear. We therefore consider a boundary on a point $x_0 = \epsilon$ and then we take the limit $\epsilon \rightarrow 0$. Such a limit is finite and the limit process may be interpreted as a renormalization process.

We will consider the holographic interpretation for the massless higher spin field. The massless mode is going to be mapped to a boundary conformal invariant operator. Concerning the massive ones, a massive KK mode in $d = p + 1$ dimensions behaves at the boundary $x_0 = \infty$ equivalent to $y \rightarrow \infty$ as

$$\Phi_m \sim x_0^{(p-1)-2s}. \quad (6.7)$$

Therefore all the massive modes for a spin field $s > (p-1)/2$ do not contribute at the boundary. In five dimensions the only massive mode that could contribute are $s = 1$ gauge and $s = 0$ scalar

fields. However, the effect of massive KK state is to introduce logarithmic divergences, which can be reabsorbed by renormalization [41]. Therefore, the important modes are only the massless ones.

The next step is to solve the HS field equations and plug back the solution into the action (6.1) in order to calculate the two-point function of HS operators of the boundary *CFT*. For this, we will assume appropriate boundary conditions, which are written as

$$\Phi_{a_1 \dots a_s}(x_0 = 0, x^a) = \phi_{a_1 \dots a_s}(x^a), \quad \Phi_{0\alpha_2 \dots \alpha_s}(x_0 = 0, x^a) = 0 \quad (6.8)$$

where Greek and Latin indices run over 5 and 4 dimensions, respectively ($\alpha_1 \dots = 0, \dots, 4$, $a_1 \dots = 1, \dots, 4$). Moreover, the solutions we are after, approach a δ -function at the boundary. As in the cases already discussed [39],[40], this can be achieved as follows. One finds first solutions which behave like δ -function at $x_0 = \infty$ in the sense that the boundary condition

$$\Phi_{a_1 \dots a_s}(x_0 \rightarrow \infty, x^a) \rightarrow \infty, \quad \Phi_{0\alpha_2 \dots \alpha_s}(x_0 \rightarrow \infty, x^a) = 0, \quad (6.9)$$

are satisfied and then use the inversion transformation (6.13) given below, for the solution at $x_0 = 0$. Using this method, one finds that $\Phi_{0\alpha_2 \dots \alpha_s}$ do not couple to $\Phi_{a\alpha_2 \dots \alpha_s}$, and by (6.9), $\Phi_{0\alpha_2 \dots \alpha_s}$ can be consistently put to zero ($\Phi_{0\alpha_2 \dots \alpha_s} = 0$). Moreover, we recall that a massless state solution can be written as

$$\Phi_{a_1 \dots a_s}(x_0, x^a) = f(x_0)\phi_{a_1 \dots a_s}(x^a), \quad (6.10)$$

where $\phi_{a_1 \dots a_s}(x^a)$ is the boundary value of the field Φ and it is transverse and traceless $\nabla^a \phi_{aa_2 \dots a_s} = \phi^a_{aa_3 \dots a_s} = 0$. Inserting this ansatz in (6.3) one find two possible solutions

$$f_1(x_0) = (2ax_0)^{2(1-s)}, \quad f_2(x_0) = (2ax_0)^2. \quad (6.11)$$

For $s > 2$, the first solution is the normalizable one and it has been discussed previously. However since we are looking for a divergent solution on the $x_0 = \infty$ boundary we will use the non normalizable one. So the solution is then

$$\Phi_{a_1 \dots a_s} = N4a^2 x_0^2 \phi_{a_1 \dots a_s}(x^a), \quad (6.12)$$

where N is a normalization factor.

We now map the point at infinity (which make the field divergent) to a point in zero with the $SO(2, 4)$ transformation

$$x^\mu \rightarrow \frac{x^\mu}{x_0^2 + |x|^2}, \quad (6.13)$$

where $|x|$ is the distance of a four dimensional point from the origin. We also introduce the function $\Omega = x_0^2 + |x|^2$. With this transformation we obtain

$$\Phi_{\alpha_1 \dots \alpha_s} = N4a^2 \frac{x_0^2}{\Omega^{s+2}} \mathcal{I}_{\alpha_1 b_1} \dots \mathcal{I}_{\alpha_s b_s} \phi^{b_1 \dots b_s}, \quad (6.14)$$

where

$$\mathcal{I}_{\mu\nu} = \eta_{\mu\nu} - \frac{2x_\mu x_\nu}{x_0^2 + |x|^2}, \quad (6.15)$$

and all indices are rise and lowered by the Euclidean metric $\eta_{\mu\nu}$.

Even if the function x_0^2/Ω^{s+2} is divergent in the point $x_0 = 0 = |x|$ is not yet actually a Dirac delta function so it cannot represent a Green function for ϕ . It is very simple to see that instead a Dirac delta function can be represented as

$$\delta^{(4)}(x^a) = \lim_{x_0 \rightarrow 0} \frac{\pi^2}{s(s+1)} \int d^4x \frac{x_0^{2s}}{\Omega^{s+2}}. \quad (6.16)$$

We can obtain a Green function then by raising $s-1$ indices on Φ and setting $N = s(s+1)/\pi^2(2a)^{2s}$. Then a generic field $\Phi(0, x^a)$ can be obtained in the limit $x_0 \rightarrow 0$ by the superposition

$$\Phi_{\alpha_1 \alpha_2 \dots \alpha_s} = \frac{s(s+1)}{\pi^2} \int d^4x' \frac{x_0^{2s}}{\Omega(x_0, |x-x'|)^{s+2}} \mathcal{I}_{\alpha_1 b_1} \mathcal{I}_{\alpha_2 b_2} \dots \mathcal{I}_{\alpha_s b_s} \phi^{b_1 \dots b_s}(x'), \quad (6.17)$$

where it has to be considered $\mathcal{I}_{ab} = \mathcal{I}_{ab}(x_0, x-x')$.

From the superposition (6.17) we can always lower and rise index in such a way that

$$\Phi^{\alpha_1 \alpha_2 \dots \alpha_s} = \frac{s(s+1)}{4a^2\pi^2} \int d^4x' \frac{x_0^4}{\Omega(x_0, |x-x'|)^{s+2}} \mathcal{I}^{\alpha_1 b_1} \mathcal{I}_{\alpha_2 b_2} \dots \mathcal{I}_{\alpha_s b_s} \phi^{b_1 \dots b_s}(x'). \quad (6.18)$$

We may also easily calculate the derivative, to be used below, which is given by

$$\partial_0 \Phi^{\alpha_1 \alpha_2 \dots \alpha_s} = \frac{s(s+1)}{a^2\pi^2} \int d^4x' \frac{x_0^3}{\Omega(x_0, |x-x'|)^{s+2}} \mathcal{I}^{\alpha_1 b_1} \mathcal{I}_{\alpha_2 b_2} \dots \mathcal{I}_{\alpha_s b_s} \phi^{b_1 \dots b_s}(x') + O(x_0^4). \quad (6.19)$$

The next step is to evaluate the action (6.1) for the field we found above. Taking into account that the action (6.1) is written as a total derivative (6.4), evaluation of (6.5) gives

$$S_B = - \int d^4x \sqrt{-g_{\text{ind}}} \Phi^{a_1 \dots a_s} \partial^0 \Phi_{a_1 \dots a_s} \quad (6.20)$$

The boundary action at $x_0 = \epsilon$ is calculated then to be

$$\begin{aligned} S_B &= - \frac{s^2(s+1)^2}{a^2\pi^4} \int d^4x \epsilon^{-3} \int d^4x' \int d^4x'' \frac{\epsilon^3}{\Omega(\epsilon, |x-x'|)^{s+2}} \mathcal{I}_{a_1 b_1} \dots \mathcal{I}_{a_s b_s} \phi^{b_1 \dots b_s}(x') \times \\ &\times \frac{\epsilon^{2s}}{\Omega(\epsilon, |x-x''|)^{s+2}} \mathcal{I}_{c_1 d_1} \dots \mathcal{I}_{c_s d_s} \phi^{d_1 \dots d_s}(x'') \eta^{a_1 c_1} \dots \eta^{a_s c_s} + O(\epsilon^{2s+1}) \end{aligned} \quad (6.21)$$

In the limit $\epsilon \rightarrow 0$, recalling the definition of the Dirac delta function (6.16) and using the fact that $\lim_{x \rightarrow 0} \mathcal{I}_{\mu\nu} = \eta_{\mu\nu}$ keeping $\epsilon \neq 0$, we obtain

$$S_B = - \frac{s(s+1)}{a^2\pi^2} \int \int d^4x d^4x' \frac{\phi^{a_1 \dots a_s}(x) \mathcal{I}_{a_1 b_1} \dots \mathcal{I}_{a_s b_s} \phi^{b_1 \dots b_s}(x')}{|x-x'|^{2s+4}}. \quad (6.22)$$

As ϕ is symmetric and traceless and $\mathcal{I}_{a_1(b_1} \mathcal{I}_{b_2)a_2}$ is completely symmetric, the action (6.22) may be rewritten as

$$S_B = - \frac{s(s+1)}{a^2\pi^2} \int \int d^4x d^4x' \frac{\hat{\phi}^{a_1 \dots a_s}(x) \mathcal{E}_{a_1 \dots a_s}^{c_1 \dots c_s} \mathcal{I}_{c_1 b_1} \dots \mathcal{I}_{c_s b_s} \hat{\phi}^{b_1 \dots b_s}(x')}{|x-x'|^{2s+4}}, \quad (6.23)$$

where $\hat{\phi}$ is any initial condition at the boundary $x_0 = 0$ and $\mathcal{E}_{a_1 \dots a_s}^{c_1 \dots c_s}$ is the projector onto totally symmetric traceless s-index tensor defined as [42]

$$\mathcal{E}_{a_1 \dots a_s}^{c_1 \dots c_s} = \frac{\delta_{(a_1}^{(c_1} \dots \delta_{a_s)}^{c_s)}}{(s!)^2} + \frac{1}{s!} \sum_{r=1}^{[s/2]} \lambda_r g_{(a_1 a_2} \dots g_{a_{2r-1} a_{2r}} g^{(c_1 c_2} \dots g^{c_{2r-1} c_{2r}} \delta_{a_{2r+1}}^{c_{2r+1}} \dots \delta_{a_s)}^{c_s)}, \quad (6.24)$$

where $[s/2]$ is the integer part of $s/2$ and

$$\lambda_r = (-1)^r \frac{1}{2^r r! (s-2r)! \prod_{k=1}^r (4+2s-2-2k)} . \quad (6.25)$$

Let us now consider a lagrangian in four dimension

$$\mathcal{L} = \mathcal{L}_{CFT} + \hat{\phi}_{a_1 \dots a_s} J^{a_1 \dots a_s} + \dots \quad (6.26)$$

where $\hat{\phi}$ is an external frozen field and $J_{a_1 \dots a_s}$ is a conserved and traceless current of dimension $(s+2)$ of the *CFT*. Then one has [43]

$$\langle J_{a_1 \dots a_s}(x) J_{b_1 \dots b_s}(x') \rangle \sim \frac{\mathcal{E}_{a_1 \dots a_s}^{c_1 \dots c_s} \mathcal{I}_{c_1 b_1} \dots \mathcal{I}_{c_s b_s}}{|x - x'|^{2s+4}} . \quad (6.27)$$

The equation (6.27) is equivalent to the kernel of (6.23) in accordance with the *AdS/CFT* correspondence.

The scenario described above, change completely when one looks at the RS model. First of all the “visible” boundary is not anymore at spatial infinity and Neumann boundary conditions must be imposed. Thanks to the boundary conditions, the fields can also be dynamical in the holographic picture and the tower of massive modes do not decay. In this way the KK modes contribute to the holographic theory switching on non conformally invariant operators on the holographic theory at the boundary. However, removing the IR brane, the conformal invariance is restored and the conformal dimension of the dual operators equal their bare dimensions.

In the following we will discuss this case for fermionic fields but the bosonic case can be directly generalized from it.

B. Holography: fermions in a Box

We will discuss here the holographic picture of higher spins fermions in the RS model on the UV brane. The tensorial structure for fermionic operators is similar to the bosonic ones, and thus, we will only consider their scaling properties. Note that the case $s = 1/2$ has been already discussed by several authors [25].

We are interested in computing the two-point function of higher-spin operators in the RS case. We will again use Poincaré coordinates with metric as in (6.6). Following closely analogous computations for the lowest spin cases in standard *AdS/CFT* and in RS, we will put the UV brane, the UV regulator, at $x_0 = 1/2a$ and the TeV brane at $x_0 = 1/\mu$. We are interested in the large N limit of the corresponding holographic theory which is equivalent of requiring a large cosmological constant or, in particular, a small x_0 .

The CFT is living in the UV boundary and as fixed source fields will be taken the left-handed HS fermionic field defined by the conditions (suppressing tensor indices for convenience)

$$\Psi_L(x_0 = \frac{1}{2a}, x^a) = \Psi_L^0(x^a), \quad \text{with} \quad \delta\Psi_L \Big|_{1/2a} = 0, \quad \Psi_L \Big|_{1/\mu} = 0, \quad (6.28)$$

whereas, the right-handed component Ψ_R will be free. The fermionic HS action is given by eq.(5.3) and its variation does not vanish as the right-handed HS fields are free on the UV boundary. Thus, we are forced to add a boundary term. In the $\gamma^\mu \Psi_{\mu \dots} = 0$ gauge, this is

$$S_{\text{boundary}} = \frac{1}{g_5^2} \int_{UV} d^4x \sqrt{-g_{ind}} \left(\frac{1}{2} \bar{\Psi}_{\alpha_1 \dots \alpha_{s-1/2}} \Psi^{\alpha_1 \dots \alpha_{s-1/2}} \right), \quad (6.29)$$

where g_5^2 is the bulk coupling constant of the fermionic gauge field $\Psi_{\alpha_1 \dots \alpha_{s-1/2}}$.

We recall that in the gauge $\gamma^\mu \Psi_{\mu \dots} = 0$, the HS fermionic field equations turn out to be

$$\not{D}\Psi_{\alpha_1 \dots \alpha_{s-1/2}} + 2as\Psi_{\alpha_1 \dots \alpha_{s-1/2}} = 0 , \quad (6.30)$$

as $\Psi_{5 \dots}$ decouples and can consistently be taken to vanish. These equations, after projecting with $1 \pm \gamma^5$ are written as

$$\begin{aligned} \partial_0 \Psi_L + (2s - \frac{5}{2}) \frac{1}{x_0} \Psi_L + \hat{\gamma}^a \partial_a \Psi_R &= 0 \\ \partial_0 \Psi_R - \frac{5}{2} \frac{1}{x_0} \Psi_R - \hat{\gamma}^a \partial_a \Psi_L &= 0 , \end{aligned} \quad (6.31)$$

where, for convenience, all tensor indices are suppressed. Let us consider a solution in the four dimensional momentum space of the type

$$\Psi_{L,R}(p, x_0) = \frac{f_{L,R}(p, x_0)}{f_{L,R}(p, 1/2a)} \Psi_{L,R}^0(p) , \quad (6.32)$$

where $\Psi_{L,R}^0(p)$ is the wave function at the UV boundary. With this separation (6.31) satisfy the equations

$$\begin{aligned} \partial_0 f_L + (2s - \frac{5}{2}) \frac{1}{x_0} f_L - p f_R &= 0 \\ \partial_0 f_R - \frac{5}{2} \frac{1}{x_0} f_R + p f_L &= 0 \\ i \not{p} \Psi_{R,L}^0 &= -p \frac{f_{R,L}(p, 1/2a)}{f_{L,R}(p, 1/2a)} \Psi_{L,R}^0 . \end{aligned} \quad (6.33)$$

It is not difficult to verify that the solution for $f_{L,R}$ using the boundary condition $\Psi_R(1/\mu) = 0$ is

$$\begin{aligned} f_L(p, x_0) &= x_0^{3-s} \left[J_{s+1/2}(p x_0) Y_{s-1/2}(p/\mu) - J_{s-1/2}(p/\mu) Y_{s+1/2}(p x_0) \right] \\ f_R(p, x_0) &= x_0^{3-s} \left[J_{s-1/2}(p x_0) Y_{s-1/2}(p/\mu) - J_{s-1/2}(p/\mu) Y_{s-1/2}(p x_0) \right] . \end{aligned} \quad (6.34)$$

It is clear that due to the field equations (5.1), the bulk HS action (5.3) vanish on shell and thus the only contributions results from the boundary term (6.29). As a result, the boundary action turns out to be

$$S_{\text{boundary}} = \frac{1}{g_5^2} \int d^4 p \bar{\Psi}_L^0(p) \Sigma(p) \Psi_L^0(-p) , \quad (6.35)$$

where, we have defined

$$\Sigma(p) = -\frac{1}{2} \frac{p}{i \not{p}} \frac{f_R(p, 1/2a)}{f_L(p, 1/2a)} . \quad (6.36)$$

Then, according to the standard nomenclature, we have that S_{boundary} is the generating functional of connected Green functions in the boundary theory. As a result, we will have

$$\langle \mathcal{O}_{a_1 \dots a_{s-1}}(p) \bar{\mathcal{O}}^{c_1 \dots c_{s-1}}(-p) \rangle = \mathcal{E}_{a_1 \dots a_s}^{c_1 \dots c_s} g_5^{-2} \Sigma(p) , \quad (6.37)$$

where the tensorial structure is encoded in $\mathcal{E}_{a_1 \dots a_s}^{c_1 \dots c_s}$. In the $a \rightarrow \infty$ limit, we get

$$\langle \mathcal{O}(p) \bar{\mathcal{O}}(-p) \rangle = \frac{-i g_5^{-2}}{2^{2s} \Gamma(s + \frac{1}{2})} \frac{\not{p}}{2a} \left(\frac{p}{2a} \right)^{2s-1} \left(\ln(p/2a) - \pi \frac{Y_{s-1/2}(p/\mu)}{J_{s-1/2}(p/\mu)} \right) . \quad (6.38)$$

In the above, we have kept only the first non-analytic term, we have ignored analytic terms and the tensorial structure has been suppressed. For Euclidean momenta $p \rightarrow ip$ and $p \gg \mu$, we get in particular

$$\langle \mathcal{O}(p) \bar{\mathcal{O}}(-p) \rangle = \frac{(-)^{s-\frac{1}{2}} g_5^{-2}}{2^{2s} \Gamma(s+\frac{1}{2})} p^{2s-1} (2a)^{-2s+2} \ln(p/2a) \quad , \quad (6.39)$$

which is what is expected for the two-point function of operators of dimension $\dim[\mathcal{O}] = s + 2$.

Note that in the bosonic sector of the dual *CFT* to a non-critical string theory with a UV cut-off, a similar structure to (6.39) arises whenever one consider a scalar field with angular momentum ℓ [44]. In that case the dimension of the operator \mathcal{O} of the dual theory is related to the partial wave of momentum ℓ of the bulk scalar field, where again the non-perturbative dimensions of the \mathcal{O} s equal their bare dimensions.

7. CONCLUSIONS

It is known, that propagation of free HS fields can consistently be defined on *AdS* spacetimes. Here, we have discussed the dynamics of such fields not in the whole of *AdS* but rather in a part of it, and in particular, in a RS background. The aim was to find the spectrum of the resulting 4D HS theory. To reach this purpose, we first specified the boundary conditions that should be satisfied by the HS fields living in the bulk of the *AdS* spacetime, and then we solved the HS field equations in the bulk of *AdS* for all spins, integer and half-integer. The resulting 4D spectrum consists of an infinite tower of massive states. In addition, there exists a massless mode for spin $s = 1$. Massless mode also exist for bosons with $s > 1$ if a boundary term is introduced. This is a boundary mass for the HS fields. Hence, with the addition of such term, the 4D spectrum consist of an infinite tower of massive as well as massless modes for all integer spins. Such mode is very crucial in the *AdS/CFT* correspondence.

For fermions the situation is similar. Here, the spectrum consists of an infinite tower of massive states and the question of massless mode depends on the boundary conditions chosen. Indeed, there are boundary conditions, which do not mix 4D left and right-handed modes and lead to massless modes of definite chirality.

Another issue we have discussed here, is the interaction problem of the HS fields. We know, from the analysis in Minkowski background, that HS fields do not have minimal consistent interactions, not even with gravity. The reason is that gauge invariance, a crucial issue for the consistent propagation of HS gauge fields, is generally lost on curved backgrounds. This is due to the appearance of the Weyl tensor in the gauge transformed HS-field equations, which cannot be cancelled, even after imposing gravitational equations. In the latter case, the Weyl contribution leads to break down of the gauge invariance on a general curved background. As a consequence, tree-level unitarity is lost and the HS interacting theory is ill defined. For the case of *AdS* spacetime, which is maximally symmetric and conformally flat (vanishing Weyl tensor), gauge invariance can be maintained. However, gravitational perturbations of the *AdS* background will remove conformal flatness and thus, HS field will continue to have inconsistent gravitational interactions on the *AdS* background. Nevertheless, we should stress here that if additional gaugings are introduced, the propagation of HS fields in curved backgrounds can be perfectly well-defined leading to their consistent interaction with gravity [5],[6],[7].

Here, we extend the proposal of [32] in the case of the *AdS* spacetime. In [32], tree-level unitarity is maintained by considering massive HS fields and introducing non-minimal interactions

with gravity. These interactions cancel hard gauge-breaking terms, although gauge invariance is still softly broken due to an explicit mass term. The theory then is lacking of any hard breaking terms at linearized level, which could violate tree-level unitarity and the latter is restored up to the Planck scale. In the case of the *AdS* background, the HS gauge fields are massless. However, *AdS* space has a scale, proportional to the cosmological constant Λ . This scale is explicitly seen in the HS-field equations and has the form of a mass term, although the fields are in fact massless (as there are two propagating helicity modes). This scale allows the introduction of non-minimal interaction terms similarly to the Minkowski case which can indeed preserve tree-level unitarity. We explicitly showed this in the case of fermionic HS fields. The analysis for bosonic fields is similar, although more complicated. It should be noted that the non-minimal coupling is non-analytic in Λ so that the flat-space limit cannot be taken. In particular it should also be noted that the no-go theorem of [45] that states the impossibility of consistent coupling between HS and spin 2 particles, is circumvented by the introduction of non-minimal interactions [46].

We have also discussed possible couplings between the HS fields and the matter living at the boundary branes. Such couplings are induced from the non-minimal interaction of HS and the Weyl tensor. We know from the analysis of the Einstein equations in spaces of codimension one branes and the Israel-Darmois junction conditions, that the discontinuities in the extrinsic curvature are proportional to the local matter distribution at the points where discontinuities appear, i.e., at the brane positions [36]. As the Weyl tensor is written in terms of the curvature tensors, there is an induced coupling between the HS fields and the extrinsic curvature. Then, this coupling can be written as a local interaction term of the HS fields and the matter living on the brane. Clearly, as the Weyl tensor is traceless, only the traceless part of the brane energy-momentum tensor can be coupled to the HS fields, which is exactly what was found.

Finally, in the last part of this work, we have discussed the *AdS/CFT* correspondence for HS fields. In this case, the HS bulk field equations are solved with Dirichlet boundary condition and then the action is evaluated on the solution. This procedure gives the two-point function of HS operators of the (unknown) boundary *CFT*. We have followed this line for bosonic HS and we indeed obtained the two-point function of boundary transverse traceless operators. The same procedure in the RS background have been followed for fermionic HS fields. In this case, the same procedure produces the two-point function for boundary operators of the resulting *CFT* on the boundary. In particular we have showed that the conformal invariance of the boundary operators in the holographic *CFT* at the boundary is restored once the IR brane is removed.

As a final comment, we should mention the potential importance of HS fields in cosmology. In particular the fact that HS fields interact with matter only gravitationally and not via gauge interactions make them possible dark-matter candidates.

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